Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713926090

Eigenvalues of propagation through liquid crystals

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To cite this Article Cuypers, F. and Pauwels, H.(1988) 'Eigenvalues of propagation through liquid crystals', Liquid Crystals, 3: 8, 1157 — 1160
To link to this Article: DOI: 10.1080/02678298808086571
URL: http://dx.doi.org/10.1080/02678298808086571

LIQUID CRYSTALS, 1988, VOL. 3, NO. 8, 1157–1160

PRELIMINARY COMMUNICATIONS

Eigenvalues of propagation through liquid crystals

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(*Received 9 March 1988; accepted 27 April 1988*)

Analytical expressions are given for the propagation constant through an homogeneous liquid crystal, following the 4 × 4 matrix formulation of Berreman. Also propagation through polarizers can be calculated analytically; it is a special case of a situation described by Teitler and Henvis.

Light propagation through a liquid crystal is complicated because the medium is anisotropic and inhomogeneous. If the z axis is chosen to be perpendicular to the glass plates, the x axis can always be chosen such that the propagation is in the xz plane (see figure 1). The director orientation of the liquid crystal is described by a twist angle ϑ and a tilt angle φ. Normally ϑ and φ are functions of z. The optical dielectric constant along the director ε_1 differs from the dielectric constant perpendicular to the director, ε_2. We can look for solutions to the wave equation of the form

$$E = E(z)\exp(j(\omega t - k_x x))$$

and

$$H = H(z)\exp(j(\omega t - k_x x))$$

where $k_x = k_0 \sin\alpha$, $k_0 = \omega/c = \omega\sqrt{\varepsilon_0\mu_0}$. The angle α is the angle of incidence in free space. Berreman [1] analyses this problem in terms of the components E_x, Z_0H_y, E_y, $-Z_0H_x$ where $Z_0 = \sqrt{\mu_0/\varepsilon_0}$. Finally the liquid crystal is divided into a sufficiently large number of slabs, in which ϑ and φ are considered constant. In such a homogeneous slab, we can look for eigenmodes for which the z dependence is of the form $\exp(-jmk_0z)$. This leads to a fourth order equation for m

$$\begin{vmatrix} \Delta_{11} - m & \Delta_{12} & \Delta_{13} & 0 \\ \Delta_{21} & \Delta_{11} - m & \Delta_{23} & 0 \\ 0 & 0 & -m & 1 \\ \Delta_{23} & \Delta_{13} & \Delta_{43} & -m \end{vmatrix} = 0, \qquad (1)$$

where the Δ_{ij} are given in equation (5) of [1] ($\varepsilon_a = \varepsilon_b = \varepsilon_2$, $\varepsilon_c = \varepsilon_1$, $\varphi \to \vartheta + \pi/2$, $\vartheta \to \pi/2 - \vartheta$, $\psi = 0$, $x = \sin\alpha$). Although this equation for m can be solved with the aid of a computer, it is much faster if analytical expressions for m are known. These can be found based on the fact that there is cylindrical symmetry around the director **n**. Indeed the two modes, for which the polarization direction

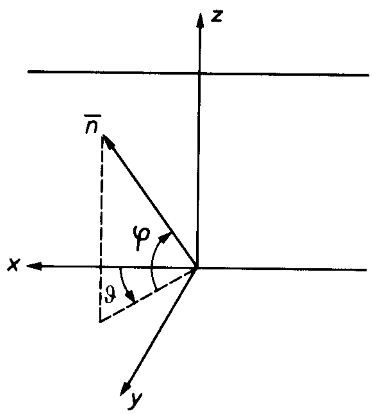

Figure 1. The twist angle ϑ and the tilt angle φ.

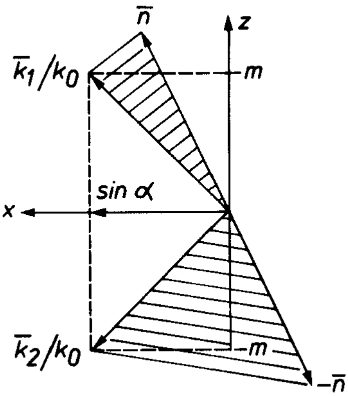

Figure 2. The ε_2 modes.

(direction of E) is perpendicular to the director and the propagation direction k, see only the dielectric constant ε_2, such that $k^2 = k_0^2\varepsilon_2$ and so

$$m^2 = \varepsilon_2 - \sin^2\alpha. \tag{2}$$

Equation (1) is therefore of the form

$$(-m^2 + \varepsilon_2 - \sin^2\alpha)(m^2 + am - b) = 0,$$

with a equal to the coefficient of m^3 which is $2\Delta_{11}$ and $b(\varepsilon_2 - \sin^2\alpha)$ as the coefficient of m^0

$$\begin{vmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} \\ \Delta_{21} & \Delta_{11} & \Delta_{23} \\ \Delta_{23} & \Delta_{13} & \Delta_{43} \end{vmatrix}.$$

Rather lengthy but straightforward calculations show that

$$a = 2\sin\alpha\,\frac{\Delta\varepsilon\cos\varphi\sin\varphi\cos\vartheta}{\varepsilon_2 + \Delta\varepsilon\sin^2\varphi},$$

$$b = \frac{\varepsilon_2(\varepsilon_1 - \sin^2\alpha) - \sin^2\alpha\Delta\varepsilon\cos^2\varphi\cos^2\vartheta}{\varepsilon_2 + \Delta\varepsilon\sin^2\varphi}$$

and thus

$$m = -\frac{a}{2} \pm \frac{1}{2}(a^2 + 4b)^{1/2}, \quad \Delta\varepsilon = \varepsilon_1 - \varepsilon_2. \qquad (3)$$

As soon as these eigenvalues are known, a computer must be used to calculate the eigenmodes, and proceed to calculate the transmission coefficient as explained in [1].

For the calculation of the transmission through a liquid crystal display, we must consider the transmission through a polarizer, through glass, through the liquid crystal, again through glass, and finally through a second polarizer. This method can also be used for the transmission through the polarizers. These have also a director with cylindrical symmetry, but which is oriented in the xy plane, and therefore φ is zero. The dielectric tensor $\boldsymbol{\varepsilon}$ is complex, since ε_1 and ε_2 contain loss factors. For positive dichroism ε_1 has the larger loss factor (such that the polarization direction ϑ_p is perpendicular to the director in the xy plane; $\vartheta_p = \vartheta + \pi/2$), for negative dichroism ε_2 has the larger loss factor and $\vartheta_p = \vartheta$. In the special case that $\varphi = 0$, equations (2) and (3) reduce to

$$m^2 = \varepsilon_2 - \sin^2\alpha, \qquad (4)$$

$$m^2 = b = \varepsilon_1 - \sin^2\alpha\left(1 + \frac{\Delta\varepsilon}{\varepsilon_2}\cos^2\vartheta\right). \qquad (5)$$

In a more general case, when there is no cylindrical symmetry around the director, the director (ε_1) and one principal axis (ε_2) are oriented in the xy plane, the third principal axis (ε_3) is along the z axis. The eigenvalues are again given by equation (1) where the Δ_{ij} can be found in equation (5) of [1] ($\varepsilon_a = \varepsilon_1$, $\varepsilon_b = \varepsilon_2$, $\varepsilon_c = \varepsilon_3$, $\vartheta = 0$, $\varphi \to \vartheta$, $\psi = 0$, $x = \sin\alpha$). This gives in our notation, with $\Delta\varepsilon = \varepsilon_1 - \varepsilon_2$

$$\left.\begin{aligned} \Delta_{12} &= 1 - \frac{\sin^2\alpha}{\varepsilon_3}, \quad \Delta_{21} = \varepsilon_1\cos^2\vartheta + \varepsilon_2\sin^2\vartheta, \\ \Delta_3 &= \Delta\varepsilon\sin\vartheta\cos\vartheta, \quad \Delta_{43} = \varepsilon_1\sin^2\vartheta + \varepsilon_2\cos^2\vartheta - \sin^2\alpha. \end{aligned}\right\} \qquad (6)$$

All other elements are zero. This leads to the equation

$$m^4 - m^2(\Delta_{12}\Delta_{21} + \Delta_{43}) + \Delta_{12}\Delta_{21}\Delta_{43} - \Delta_{23}\Delta_{12}\Delta_{23} = 0, \qquad (7)$$

or

$$m^4 - am^2 + b = 0,$$

$$a = \varepsilon_1 + \varepsilon_2 - \frac{\sin^2\alpha}{\varepsilon_3}(\varepsilon_3 + \varepsilon_1\cos^2\vartheta + \varepsilon_2\sin^2\vartheta),$$

$$b = [\varepsilon_1\varepsilon_2 - \sin^2\alpha(\varepsilon_1\cos^2\vartheta + \varepsilon_2\sin^2\vartheta)]\left(1 - \frac{\sin^2\alpha}{\varepsilon_3}\right).$$

Again this equation can easily be solved analytically. Teitler and Henvis [2] already found this result for an asymmetric tensor. We can easily verify that for $\varepsilon_3 = \varepsilon_2$, equation (7) leads to equations (4) and (5).

One of the authors, F. Cuypers is supported by the National Fund for Scientific Research of Belgium.

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